The performance of AM and FM receivers

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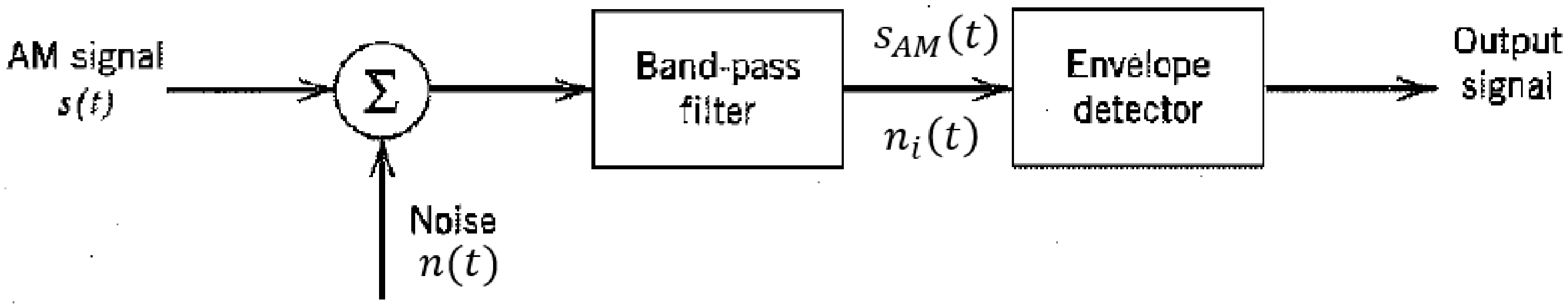
The performance of AM receivers using Envelop Detection



- In a full AM signal, both sidebands and the carrier wave are transmitted, as shown by

$$s_{AM}(t) = A_c[1 + k_{AM}m(t)]cos2\pi f_c t$$

- Assume $\overline{m(t)} = 0, \beta_{AM} \leq 1$
- After band-pass filter, the narrow band noise $n_i(t)$ is modeled as white Gaussian noise of zero mean and power spectral density $N_0/2$

$$n_i(t) = n_c(t)cos2\pi f_c - n_s(t)sin2\pi f_c t$$

The performance of AM receivers using Envelop Detection

- Then we have power of signal and noise respectively

$$S_i = \overline{s_m^2(t)} = \frac{A_c^2}{2} + \frac{A_c^2 \overline{m^2(t)}}{2}, N_i = \overline{n_i^2(t)} = N_0 W$$

- B is bandwidth of band-pass filter. The input SNR is $\frac{S_i}{N_i} = \frac{A_c^2 + A_c^2 {k_{AM}}^2 \overline{m^2(t)}}{2N_0 W}$
- The input of demodulator is

$$s_{AM}(t) + n_i(t) = \{A_c[1 + m(t)] + n_c(t)\}cos2\pi f_c t - n_s(t)sin2\pi f_c t$$

$$= E(t)\cos[2\pi f_c t + \varphi(t)]$$

- where

$$E(t) = \sqrt{\{A_c[1 + m(t)] + n_c(t)\}^2 + n_s^2(t)}$$

$$\varphi(t) = \arctan[\frac{n_s(t)}{A_c[1 + m(t)] + n_c(t)}]$$

- In order to simplify the analysis, we consider two special case.

The performance of AM receivers using Envelop Detection

- (a)Large SNR
- It means

$$A_c[1+m(t)] \gg \sqrt{n_c^2(t)+n_s^2(t)}$$

- Thus

$$E(t) = \sqrt{A_c^2[1+m(t)]^2 + 2A_c[1+m(t)]n_c(t) + n_c^2(t) + n_s^2(t)}$$

$$\approx \sqrt{A_c^2[1+m(t)]^2 + 2A_c[1+m(t)]n_c(t)}$$

$$\approx A_c[1+m(t)]\sqrt{1+\frac{2n_c(t)}{A_c[1+m(t)]}}$$

$$\approx A_c[1+m(t)]\left[1+\frac{n_c(t)}{A_c[1+m(t)]}\right]$$

$$= A_c[1+m(t)] + n_c(t)$$

Since

$$\sqrt{(1+x)} \approx 1+\frac{x}{2},\ |x| \ll 1$$

The performance of AM receivers using Envelop Detection

- After eliminating DC component, we have

$$S_o = \overline{s_m^2(t)} = A_c^2\overline{m^2(t)},\ N_o = \overline{n_c^2(t)} = \overline{n_c^2(t)} = N_0 W$$

- The output SNR is $\frac{S_o}{N_o} = \frac{A_c^2 {k_{AM}}^2 \overline{m^2(t)}}{2N_0 W}$
- Then we have $G_{AM} = \frac{S_o/N_o}{S_i/N_i} = \frac{A_c^2 {k_{AM}}^2 \overline{m^2(t)}}{A_c^2 + A_c^2 {k_{AM}}^2 \overline{m^2(t)}} = \frac{\overline{m^2(t)}}{1 + {k_{AM}}^2 \overline{m^2(t)}} < 1$
- Since $\beta_{AM} \leq 1$. And if $m(t) = A_m cos 2\pi f_m t$, $\overline{m^2(t)} = \frac{1}{2} A_m^2$

$$G_{AM} = \frac{\mu^2}{2+\mu^2}, \mu = k_{AM} A_m$$

- When 100% modulation, which means $\mu = 1$, we get 1/3. Thus we know that envelop detector lowers the SNR.

The performance of AM receivers using Envelop Detection

- (a)Small SNR
- It means

$$A_c[1+m(t)] \ll \sqrt{n_c^2(t)+n_s^2(t)}$$

- Then

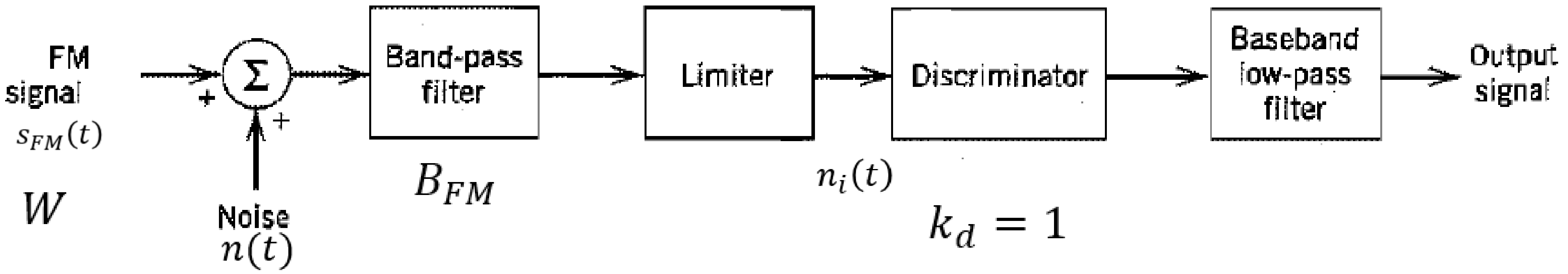
$$E(t)=\sqrt{A_c^2[1+m(t)]^2+2A_c[1+m(t)]n_c(t)+n_c^2(t)+n_s^2(t)}$$
$$\approx \sqrt{2A_c[1+m(t)]n_c(t)+n_c^2(t)+n_s^2(t)}$$
$$=\sqrt{[n_c^2(t)+n_s^2(t)]\{1+\frac{2n_c(t)A_c[1+m(t)]}{n_c^2(t)+n_s^2(t)}\}}$$
$$=R(t)\sqrt{1+\frac{2A_c[1+m(t)]}{R(t)}cos\theta(t)} \approx R(t)+A_c[1+m(t)]\,cos\theta(t)$$

- Where

$$R(t)=\sqrt{n_c^2(t)+n_s^2(t)},\theta(t)=\arctan\left[\frac{n_s(t)}{n_c(t)}\right]$$

Now the signal becomes part of noise and the system will no longer work.

The performance of FM receivers



- The incoming FM signal is defined as,

$$s_{FM}(t) = A_c \cos[2\pi f_c t + 2\pi k_{FM} \int m(\tau) d\tau]$$

- Still assume $\overline{m(t)} = 0$
- Then we have power of signal and noise respectively

$$S_i = \frac{A_c^2}{2}, N_i = \overline{n_i^2(t)} = N_0 W$$

- B is bandwidth of band-pass filter. The input SNR is $\frac{S_i}{N_i} = \frac{A_c^2}{2N_0 W}$

The performance of FM receivers

- (a)Large SNR
- Ignore the interaction between signal and noise under large SNR.
- Assume $n_i(t) = 0$,

$$m_o(t) = k_d k_{FM} m(t)$$

k_d is discriminator index, let it be 1. Thus we have

$$S_o = \overline{m_o^2(t)} = (k_{FM})^2 \overline{m^2(t)}$$

- Assume $\mathrm{m}(t) = 0$, the input of demodulator is

$$A_c cos2\pi f_c t + n_i(t) = [A_c + n_c(t)] cos2\pi f_c - n_s(t) sin2\pi f_c t$$
$$= A(t)\cos[2\pi f_c + \varphi(t)]$$

Envelop $A(t) = \sqrt{[A_c + n_c(t)]^2 + n_s^2(t)}$

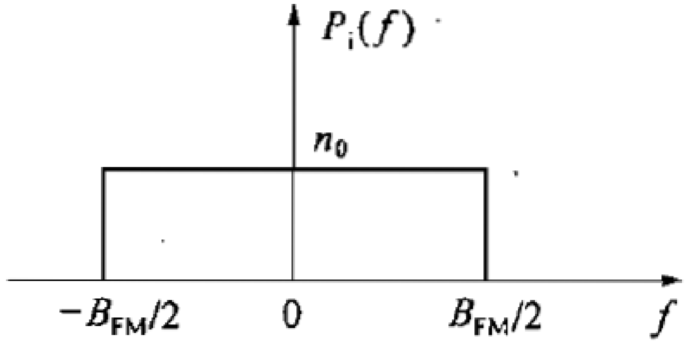
Since SNR is large, the phase

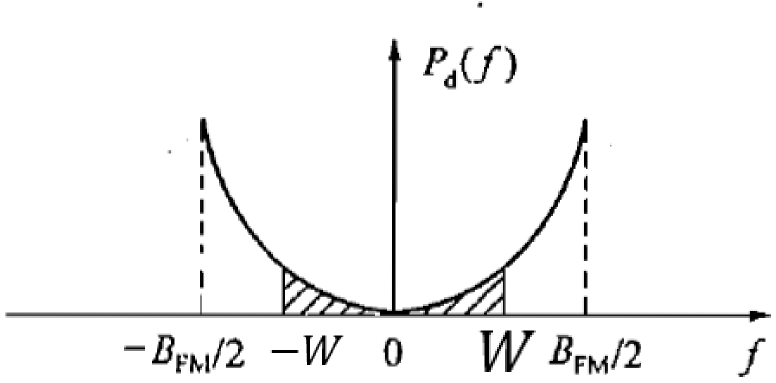
$$\varphi(t) = arctan\frac{n_s(t)}{A_c + n_c(t)} \approx arctan\frac{n_s(t)}{A_c} \approx \frac{n_s(t)}{A_c}$$

The performance of FM receivers

- Thus the output noise is

$$n_d(t) = \frac{d\varphi(t)}{dt} = \frac{1}{2\pi A_c}\frac{dn_s(t)}{dt}$$





- The power spectrum density is

$$P_d(f) = (\frac{k_d}{A_c})^2|H(f)|P_s(f) = (\frac{1}{A_c})^2 f^2 N_0, |f| < \frac{B_{FM}}{2}$$

- So we have

$$N_0 = \int_{-W}^{W} P_d(f)df = \int_{-W}^{W} (\frac{1}{A_c})^2 f^2 N_0 df = \frac{2N_0W^3}{3A_c^2}$$

The performance of FM receivers

- SNR at the output

$$\frac{S_O}{N_O} = \frac{3A_c^2 k_{FM}^2 \overline{m^2(t)}}{2N_0 W^3}$$

Then

$$G_{FM} = \frac{3k_{FM}^2 \overline{m^2(t)}}{W^2}$$

- Example : Single-tone Modulation
- The modulated FM signal is defined by

$$s_{FM}(t) = A_c \cos[2\pi f_c t + m_f sin2\pi f_m t]$$

Where $m_f = \frac{k_{FM}}{\omega_m} = \frac{\Delta\omega}{\omega_m} = \frac{\Delta f}{f_m}$

So we get

$$m(t) = \frac{\Delta f}{k_{FM}} cos2\pi f_m t$$

The performance of FM receivers

And

$$\overline{m^2(t)} = \frac{(\Delta f)^2}{2k_{FM}^2}$$

We have

$$\frac{S_O}{N_O} = \frac{3A_c^2(\Delta f)^2}{4N_0W^3} = \frac{3A_c^2\beta^2}{4N_0W}$$

- Where $\beta = \Delta f/W$ is modulation index

$$G_{FM} = \frac{3\beta^2}{2}$$

- For an AM system operating with a single-tone signal and 100% modulation,

$$G_{AM} = \frac{1}{3}$$

- We just need to adjust $\beta \geq 0.5$, there will be $G_{FM} > G_{AM}$